Observable signals in a string inspired axion-dilaton background and Randall-Sundrum scenario

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Abstract

Rotation angle of the plane of polarization of the distant galactic radio waves has been estimated in a string inspired axion-dilaton background. It is found that the axion,dual to the field strength of the second rank massless antisymmetric Kalb-Ramond field in the string spectrum, produces a wavelength independent optical rotation which is much larger than that produced by the dilaton. Detection of such rotation has been reported in some recent cosmological experiments. The observed value has been compared with our estimated theoretical value following various cosmological constraints. The effects of warped extra dimensions in a braneworld scenario on such an optical rotation have been investigated.

1   Introduction

Despite several theoretical successes, string theory is yet to make any contact with the observable world. The low energy field theory limit of a string theory followed by suitable dimensional reduction yields the four dimensional action for the supergravity (SUGRA) multiplet coupled to super-Yang Mills. Some generic features of such an action may be explored for some indirect signal of a string inspired supergravity model. Two main areas to look for such signature of string theory are the cosmological/astrophysical experiments and high energy collider experiments.

This work aims to relate a characteristic feature of a string inspired model to a much studied astrophysical phenomenon namely the rotation of the plane of polarization of distant galactic radiations[1]. It is well known,in the name of Faraday effect, that the plane of polarization of a plane polarized electromagnetic wave rotates in presence of a magnetic field. The rotation angle is directly proportional to the square of the wavelength of the radiation. Extensive studies of this have been done in astrophysical[2] as well as other experiments[3]. However some astrophysical data analysis[4]revealed a small additional wavelength independent rotation of the plane of polarization over and above the usual Faraday rotation. A phenomenological model was immediately proposed[5] where a pseudo scalar field was coupled to the electromagnetic field for a possible explanation. Such a model indeed can successfully explain such

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rotation of the plane of polarization of a plane polarized radiation. This possibility has also been pointed out in several other works where various effects of axion(a pseudo-scalar)-electromagnetic coupling have been investigated in details[6]. In all these models the coupling term of the axion and electromagnetic field has a free coupling parameter which can be fixed from experimental observations. It was however shown [7]in the context of a string inspired low energy models that such an axion-electromagnetic (em) coupling appears naturally from the consistency requirement like gauge anomaly cancellation. Moreover the coupling parameter in such a theory gets determined from the theory itself. The axion-em coupling parameter in the higher dimensional theory is given by the inverse of Planck mass whereas the effective four dimensional coupling parameter in the four dimensional effective action depends on the type of compactification used to compactify the extra dimensions.

The low energy four dimensional effective field theory action of string theory [8, 9, 10], namely the supergravity action, contains two massless fields, viz., a second rank antisymmetric tensor field (Kalb-Ramond field [11]) from the NS-NS sector of the underlying string theory as well as a scalar field called dilaton. The third rank field strength corresponding to the the Kalb-Ramond (KR) field is dual to a pseudo scalar in four dimension called axion. Various consequences of the presence of such a axion background in a curved spacetime on various physical phenomena have been investigated in view of possible indirect evidences of string theory at low energies[12, 13, 14]. The interpretation of the KR field [15, 7] as a torsion in the background spacetime [16], inevitably implies a study of electromagnetism in torsion backgrounds. In this context, the gauge $U(1)$ Chern-Simons term that appears naturally on account of gauge anomaly-cancellation in the supergravity theory plays a crucial role in establishing a gauge-invariant coupling of the KR field (or, torsion) with the electromagnetic field [7, 17]. Certain physically observable phenomena, especially in the cosmological scenario, may result from the KR-electromagnetic coupling through the $U(1)$ Chern-Simons term. One such phenomenon of particular interest, argued to be induced by the KR field [18, 19, 20], is a frequency-independent cosmic optical rotation of the plane of polarization of linearly polarized synchrotron radiation from high redshift galaxies [21]-[4]. Other important aspects of gauge-invariant Einstein-KR-electromagnetic coupling include the effects on the electric-magnetic orthogonality, energy-conservation equation and also on the electric-magnetic duality symmetry [22].

In this work we first take a four dimensional dilaton-axion-em coupled theory and estimate a theoretical bound on the value of the angle of rotation of the plane of polarization from known cosmological constraints. Subsequently we analyze this phenomenon in theories with extra dimensions. Such theories have drawn considerable attention both in the arena of high energy phenomenology as well as cosmology. Two of the most promising models are due to Arkani-Hamed, Dimopoulos and Dvali (ADD) [23] and Randall and Sundrum (RS) [24]. Both these scenarios claim to solve the well known fine tuning

problem of the Higgs mass against large radiative correction because of the gauge hierarchy. While the ADD model assumes the existence of large extra dimensions, the RS model proposes a four dimensional warped spacetime solution in a five dimensional bulk AdS spacetime. Since both these models relate Planck scale to TeV scale , some signals of them are expected even in the forthcoming TeV scale collider. Our focus in this paper however centres on a cosmological scenario where even a tiny signal can grow to a measurable value over a large cosmological length scales.We therefore re-examine the phenomenon of the rotation of the plane of polarization in presence extra compact dimensions. In particular we investigate this issue in the context of RS scenario where we show that the axion-electromagnetic coupling (which depends on the compactification scheme used to compactify the extra dimensions) changes drastically in the RS compactification resulting into some interesting consequences.

This paper addresses the above features as follows: in section 2, we explore the effective four dimensional Einstein-Maxwell-Kalb-Ramond-dilaton coupling from a five dimensional model. We study both the compactification schemes, viz., Kaluza-Klein dimensional reduction on $\mathbf{S^1}$ and RS type compactification on $\mathbf{S^1/Z_2}$. In section 3, we work out the effective field equations for the $U(1)$ gauge field as well as the equations of motion for the KR axion (the hodge dual pseudo-scalar of the third rank antisymmetric KR field strength tensor in four dimensions) and the dilaton, in a cosmological scenario. The KR field induced phenomena of cosmic optical activity (leading to a birefringence in background spacetime) has been analyzed in detail in section 4. We consider a homogeneous and isotropic cosmological background of Friedmann-Robertson-Walker (FRW) type and study the optical rotation in four dimensions when both dilaton and axion are present as 4+1 dimensional 'bulk' fields. The $U(1)$ gauge field is also considered to be existing in the bulk. Precise estimates of the optical rotation angle is determined in the cases corresponding to both the compactification schemes discussed above. We also compute the maximum KR field energy density from the maximum bound on the optical rotation angle given by observational data for radiation from distant galaxies, and compare the result both with the Cosmic Microwave Background (CMB) energy density and with the present dark matter density of the Universe.

2   Effective four dimensional Einstein-Maxwell-Kalb-Ramond-dilaton theory from a five dimensional 'bulk' model

Let us consider a general five dimensional action for Einstein-Maxwell-Kalb-Ramond-dilaton (EMKRD) coupling in the Einstein frame

$$S_{eff} = \int d^5x\sqrt{-G}\left[M^3\left(R^{(5)} - \frac{1}{2}\partial_M\Phi\partial^M\Phi\right) - \frac{1}{12}e^{-2\Phi}H_{MNL}H^{MNL} - e^{-\Phi}F_{MN}F^{MN}\right] \quad (1)$$

where $R^{(5)}$ is the five dimensional scalar curvature corresponding to the metric G_{MN}; M is the five dimensional Planck mass; Φ is the massless dilaton field; $F_{MN} = \partial_{[M}A_{N]}$ is the field strength of a $U(1)$

gauge field A_N; and $H_{MNL} = \partial_{[M}B_{NL]} + 2M^{-3/2}A_{[M}F_{NL]}$ is the strength of the KR two form B_{MN} augmented with $U(1)$ Chern-Simons term $A_{[M}F_{NL]}$. All the fields are supposed to live in the entire 'bulk', i.e., in the usual 3+1 dimensional spacetime (x) as well as the extra compact space (y).

In order to have a four dimensional effective theory with Kalb-Ramond, dilaton and the $U(1)$ gauge fields, we resort to two specific schemes of compactification as follows:

2.1 Scheme I : Kaluza-Klein dimensional reduction on $\mathbf{S^1}$

In this type of compactification any field $\Psi(x, y)$ is decomposed in the form

$$\Psi(x,y) \;=\; \frac{1}{\sqrt{L}} \sum_{n=0}^{\infty} \psi^n(x) \exp\left[\frac{2\pi i n y}{L}\right] \tag{2}$$

where L is the radius of compactification. For the graviton, Kalb-Ramond and $U(1)$ gauge fields, various scalars and vector moduli fields appear by virtue of the above scheme. The dilaton Φ however does not give rise to any such modular field. In fact, it is convenient to shift the dilaton by a constant volume factor, so that in the effective four dimensional theory the dilaton is given by $\phi = \Phi - \ln V$. One can now write the four dimensional low energy effective action for the massless modes as

$$S \;=\; \int d^4x \sqrt{-g} \left[M_p^2 \left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \right) - \frac{1}{12}e^{-2\phi}H_{\mu\nu\lambda}H^{\mu\nu\lambda} \;-\; e^{-\phi}F_{\mu\nu}F^{\mu\nu} \right] \tag{3}$$

where R is the four dimensional scalar curvature corresponding to the four-metric $g_{\mu\nu}$; the four dimensional Planck mass M_p is related to the five dimensional Planck mass M as: $M_p^2 = M^3 L$. $H_{\mu\nu\lambda} = \partial_{[\mu}B_{\nu\lambda]} + 2M_p^{-1}A_{[\mu}F_{\nu\lambda]}$ is the KR field strength augmented with $U(1)$ Chern-Simons term. The massive modes, which have masses inversely related to the radius of the compact dimension, i.e. L, may be ignored as long as the radius is small $\sim M_p$, but can be significant in theories with large extra dimensions [23].

2.2 Scheme II : Compactification on $\mathbf{S^1/Z_2}$

Let us now resort to an alternative compactification scheme similar to that in the Randall-Sundrum (RS) model. In the minimal version of such a model, described in a five dimensional AdS spacetime, the extra coordinate y is compactified on a $\mathbf{S^1/Z_2}$ orbifold. Two branes, viz., the hidden and visible branes, are located at two orbifold fixed points $y = 0$ and $y = \pi$ respectively. The line element of the corresponding background

$$ds^2 \;=\; e^{-2\sigma(y)}\eta_{\alpha\beta}dx^\alpha dx^\beta \;+\; r_c^2 dy^2 \tag{4}$$

describes a non-factorisable geometry with an exponential warping over a flat $(\eta_{\alpha\beta})$ four dimensional part. The warp factor is given in terms of the parameter $\sigma = \kappa r_c|y|$, where r_c is the compactification radius and κ is of the order of the higher dimensional Planck scale M. The four dimensional Planck mass

M_p is related to the five dimensional Planck mass M as:

$$M_p^2 = \frac{M^3}{\kappa} \left(1 - e^{-2\kappa r_c \pi}\right).$$

Starting from the five dimensional free action for the dilaton field Φ:

$$S_\Phi = -\frac{M^3}{2} \int d^5 x \sqrt{-G}\, G^{MN} \partial_M \Phi \partial_N \Phi \qquad (5)$$

we apply a Kaluza-Klein (KK) decomposition of the dimensionless dilaton field $\Phi(x,y)$, viz.,

$$\Phi(x,y) = \sum_{n=0}^{\infty} \phi^n(x)\, \zeta^n(y) \qquad (6)$$

Now following the usual procedure of analyzing the bulk fields in RS scenario, we obtain the solution for the massive modes

$$\zeta_n(y) \approx \left[\frac{\sqrt{\kappa r_c}\, e^{-\kappa r_c \pi}}{J_3\left(\frac{m_n}{\kappa} e^{\kappa r_c \pi}\right)}\right] e^{2\kappa r_c y}\, J_2\left(\frac{m_n}{\kappa} e^{\kappa r_c y}\right) \qquad (7)$$

For the zero mass mode $(n = 0, m_n = 0)$, however, the solution of the equation is given by

$$\zeta_0(y) = c_0 + \frac{c_1}{\kappa r_c} e^{4\kappa r_c |y|} \qquad (8)$$

The condition of self-adjointness of course leads to $c_1 = 0$ and leaves the scope of only a constant solution for ζ_0. Using the normalization condition, one finally obtains

$$\zeta_0 = \sqrt{\kappa r_c}\, e^{-\kappa r_c \pi} \qquad (9)$$

which indicates that the massless mode of the dilaton field is suppressed by a huge warp factor on the visible brane. The situation is similar to the RS compactification of the free Kalb-Ramond Lagrangian, for which the massless KR projection is suppressed by the same exponential factor [25]. The massive modes, however, have different solutions in this case.

As we are interested in a low energy effective four dimensional compactified version of the Einstein-Maxwell-KR-dilaton theory, we take into account only the massless modes of the projections of all the bulk fields on the visible brane. Now, in the bulk the dilaton has exponential couplings with both the KR field and the $U(1)$ gauge field. Therefore, when one considers only the massless dilaton mode which carries an enormous suppression factor as shown above, it is clear that the exponential dilaton couplings can be very well approximated as $e^{-\phi} \sim 1$, in the four dimensional effective action on the visible brane. In other words, restricting one's attention to the effects at low energies, we are in a sense led to a scenario in which the Einstein-Maxwell-KR Lagrangian is supplemented only with the free dilaton Lagrangian $\frac{1}{2}(\partial\phi)^2$ and there is virtually no exponential dilaton coupling with either the KR field or the $U(1)$ gauge field. The RS compactification of the free Kalb-Ramond Lagrangian has already been studied extensively in [25]. Similar kind of compactification for the bulk $U(1)$ gauge field has been studied in [26] and also

elucidated in [27] in the context of cosmic optical activity. We write down the essential results obtained therein as follows:

Decomposing the bulk KR field $B_{\mu\nu}(x, y)$ and the bulk $U(1)$ gauge field $A_\mu(x, y)$ respectively as

$$B_{\mu\nu}(x,y) \;=\; \frac{1}{\sqrt{r_c}} \sum_{n=0}^{\infty} B_{\mu\nu}^n(x)\chi^n(y) \;; \qquad A_\mu(x,y) \;=\; \frac{1}{\sqrt{r_c}} \sum_{n=0}^{\infty} A_\mu^n(x)\psi^n(y) \tag{10}$$

and performing an analysis similar to that shown above for the dilaton, one finds the solution for the massless mode of the KR field [25] and that for the $U(1)$ gauge field [26] as

$$\chi^0 \;=\; \sqrt{\kappa r_c}\, e^{-\kappa r_c \pi} \;; \qquad \psi^0 \;=\; \frac{1}{\sqrt{2\pi}}. \tag{11}$$

Finally, taking everything together and remembering that the three-tensor H_{MNL} in the bulk is given by the strength of the KR field B_{MN} augmented with the $U(1)$ Chern-Simons term $A_{[M}F_{NL]}$, one obtains the RS compactified four dimensional low energy effective EMKRD action as

$$S \;=\; \int d^4x \sqrt{-g} \left[M_p^2 \left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \right) - \frac{1}{12}e^{-2\zeta_0\phi}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - e^{-\zeta_0\phi}F_{\mu\nu}F^{\mu\nu} \right] \tag{12}$$

where $\zeta_0 = \sqrt{\kappa r_c}\, e^{-\kappa r_c \pi} \sim 10^{-17}$ for $\kappa r_c = 12$ (chosen to solve the naturalness problem in the standard model), and we denote the massless mode ϕ^0 of the dilaton as ϕ which we call the massless four dimensional dilaton field. Similarly we also omit the superscript "0" in the massless modes $B_{\mu\nu}^0(x)$ and $A_\mu^0(x)$ for the KR field and the $U(1)$ gauge field respectively. The three tensor $H_{\mu\nu\lambda}$ is now given by

$$H_{\mu\nu\lambda} \;=\; \partial_{[\mu}B_{\nu\lambda]} + 2\sqrt{\frac{\kappa}{M_p^3}}\, e^{\kappa r_c \pi}A_{[\mu}F_{\nu\lambda]} \tag{13}$$

which shows a huge enhancement ($\sim e^{\kappa r_c \pi}$) in the coupling between the KR field strength (dual to the string axion) and the $U(1)$ gauge field because of the Chern-Simons term. This enhancement is a major outcome of the RS compactification implemented on the KR-Maxwell part of the five dimensional action (1) as has been shown explicitly in [27].

3 Effective Field equations in a cosmological scenario

In presence of standard cosmological matter, for example perfect fluid, we write down a general four dimensional low energy effective EMKRD action as

$$S \;=\; \int d^4x \sqrt{-g} \left[M_p^2 \left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \right) - \frac{1}{12}e^{-2\alpha\phi}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - e^{-\alpha\phi}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_m \right] \tag{14}$$

where

$$H_{\mu\nu\lambda} \;=\; \partial_{[\mu}B_{\nu\lambda]} + \frac{2\beta}{M_p}A_{[\mu}F_{\nu\lambda]} \;; \tag{15}$$

is the modified KR field strength, $\mathcal{L}_m$ is the cosmological matter Lagrangian density, and α and β are two free dimensionless parameters whose values are specified according to the two schemes of compactification respectively as:

$$\alpha \;=\; \beta \;=\; 1 \;:\qquad\qquad\qquad\qquad\qquad\text{Scheme I}$$

$$\alpha \;=\; \sqrt{\kappa r_c}\, e^{-\kappa r_c \pi} \ll 1,\; \beta \;=\; \sqrt{\frac{\kappa}{M_p}}\, e^{\kappa r_c \pi} \gg 1 \;:\qquad \text{Scheme II.}\qquad(16)$$

In terms of the massless and dimensionless pseudo scalar 'axion' field ξ, which is related to $H^{\mu\nu\lambda}$ via the duality

$$H^{\mu\nu\lambda} \;=\; M_p\, e^{2\alpha\phi}\, \epsilon^{\mu\nu\lambda\rho}\, \partial_\rho \xi \;,\qquad(17)$$

the modified Maxwell (electromagnetic) equations are given by

$$(a)\qquad \nabla_\mu\left(e^{-\alpha\phi} F^{\mu\nu} \;+\; \beta\,\xi\, {}^*F^{\mu\nu}\right) = 0$$

$$(b)\qquad \nabla_\mu\, {}^*F^{\mu\nu} \;=\; 0\qquad\qquad\qquad(18)$$

where ∇_α stands for the covariant derivative and ${}^*F_{\alpha\beta} = \epsilon_{\alpha\beta\mu\nu} F^{\mu\nu}$ is the Hodge dual of the electromagnetic field strength $F^{\mu\nu}$.

The coupled equations of motion for the axion and dilaton are expressed as

$$(a)\qquad \nabla_\mu\left(e^{2\alpha\phi} \partial^\mu \xi\right) = \frac{\beta}{M_p^2}\, F_{\mu\nu}\, {}^*F^{\mu\nu}$$

$$(b)\qquad \nabla_\mu \partial^\mu \phi \;=\; \alpha\, e^{2\alpha\phi}\, \partial_\mu \xi \partial^\mu \xi \;-\; \frac{\alpha}{M_p^2}\, e^{-\alpha\phi}\, F_{\mu\nu} F^{\mu\nu}.\qquad(19)$$

We consider the spatially flat homogeneous and isotropic Robertson-Walker line element,

$$ds^2 \;=\; a^2(\eta)\left[-d\eta^2 \;+\; d\mathbf{x}^2\right]\qquad(20)$$

where $a(\eta)$ is the scale factor; and the conformal time η is related to the cosmic time t through the relation $dt = a(\eta)d\eta$.

The electromagnetic field equations take the form

$$(a)\qquad \nabla\cdot\tilde{\mathbf{E}} \;=\; \alpha\, \nabla\phi\cdot\tilde{\mathbf{E}} \;-\; \beta\, e^{\alpha\phi}\, \nabla\xi\cdot\tilde{\mathbf{B}}$$

$$(b)\qquad \dot{\tilde{\mathbf{E}}} \;-\; \nabla\times\tilde{\mathbf{B}} \;=\; \alpha\left(\dot{\phi}\,\tilde{\mathbf{E}} \;-\; \nabla\phi\times\tilde{\mathbf{B}}\right) \;-\; \beta\, e^{\alpha\phi}\left(\dot{\xi}\tilde{\mathbf{B}} \;+\; \nabla\xi\times\tilde{\mathbf{E}}\right)$$

$$(c)\qquad \nabla\cdot\tilde{\mathbf{B}} \;=\; 0$$

$$(d)\qquad \dot{\tilde{\mathbf{B}}} \;+\; \nabla\times\tilde{\mathbf{E}} \;=\; 0\qquad\qquad(21)$$

where the overhead dot stands for partial differentiation with respect to η, and we denote

$$\tilde{\mathbf{E}}(\eta,\mathbf{x}) \;\equiv\; a^2(\eta)\,\mathbf{E}(\eta,\mathbf{x}) \;;\qquad \tilde{\mathbf{B}}(\eta,\mathbf{x}) \;\equiv\; a^2(\eta)\,\mathbf{B}(\eta,\mathbf{x})\qquad(22)$$

E and **B** being the usual electric and magnetic fields respectively.

The equations of motion for the axion and the dilaton reduce to

$$(a) \quad \ddot{\xi} + 2\frac{\dot{a}}{a}\dot{\xi} - \nabla^2\xi + 2\alpha\left(\dot{\phi}\,\dot{\xi} - \nabla\phi\cdot\nabla\xi\right) = \frac{2\beta}{M_p^2\,a^4}e^{-2\alpha\phi}\tilde{\mathbf{E}}\cdot\tilde{\mathbf{B}}$$

$$(b) \quad \ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - \nabla^2\phi - \alpha\,e^{2\alpha\phi}\left(\dot{\xi}^2 - |\nabla\xi|^2\right) = \frac{2\alpha}{M_p^2\,a^4}e^{-\alpha\phi}\left(\tilde{\mathbf{E}}^2 - \tilde{\mathbf{B}}^2\right). \tag{23}$$

It follows from the electromagnetic field equations (21) that

$$\partial_\eta\left(e^{-\alpha\phi}\tilde{\mathbf{E}}\cdot\tilde{\mathbf{B}}\right) = e^{-\alpha\phi}\left(\tilde{\mathbf{B}}\cdot\nabla\times\tilde{\mathbf{B}} - \tilde{\mathbf{E}}\cdot\nabla\times\tilde{\mathbf{E}}\right) - \beta\left(\dot{\xi}\,|\tilde{\mathbf{B}}|^2 + \nabla\xi\cdot\tilde{\mathbf{E}}\times\tilde{\mathbf{B}}\right) \tag{24}$$

which shows that the vectors $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ are <u>not</u> in general mutually orthogonal. This is in sharp contrast with the plane wave nature of the free electric and magnetic fields $(\tilde{\mathbf{E}},\,\tilde{\mathbf{B}} \sim e^{i(k\eta - \mathbf{k}\cdot\mathbf{x})})$ for $\phi = \xi = 0$. This leads to an optical anisotropy (and/or an effective magnetization) in the background spacetime in presence of the dilaton and the axion, which is manifested observationally through a cosmic optical rotation of the plane of polarization of radiation from distant galactic sources. Such an optical activity, which makes a hitherto isotropic medium birefringent, is in fact entirely due to the typical axionic coupling with the electromagnetic field through the $U(1)$ Chern-Simons term as shown in [18, 22]. The dilaton on its own does not produce the birefringence, however, there may be a substantial change in the amount of axion-induced optical rotation in presence of the dilaton. In the following section we study this optical activity in presence both axion and dilaton and determine the amount of optical rotation of the plane of polarization of galactic radiation in both flat background and spatially flat Friedmann-Robertson-Walker (FRW) background.

Over the large cosmological scales we take into account, in our subsequent analysis, only the temporal variations of the axion and dilaton fields, i.e., $\xi = \xi(\eta)$, $\phi = \phi(\eta)$, whence the equations (23) take the forms (on dropping the $\mathcal{O}(M_p^{-2})$ terms)

$$(a) \quad \partial_\eta\left(a^2\,e^{2\alpha\phi}\,\dot{\xi}\right) = 0$$

$$(b) \quad \partial_\eta\left(a^2\,\dot{\phi}\right) = \alpha\,e^{2\alpha\phi}\,a^2\,\dot{\xi}^2 \tag{25}$$

These can be solved exactly to obtain

$$\xi(\eta) = \frac{1}{\alpha}\tanh\left[\alpha\,h\int^\eta\frac{d\eta'}{a^2(\eta')}\right]\;;\qquad \phi(\eta) = \frac{1}{\alpha}\ln\left\{\cosh\left[\alpha\,h\int^\eta\frac{d\eta'}{a^2(\eta')}\right]\right\} \tag{26}$$

where the constant parameter h determines the rates of evolution of the free axion and dilaton fields.

4   Cosmic Optical Activity in presence of axion and dilaton

The electromagnetic wave equations in presence of the KR axion and the dilaton are given by

$$(a) \quad \Box\tilde{\mathbf{B}} \equiv \ddot{\tilde{\mathbf{B}}} - \nabla^2\tilde{\mathbf{B}} = \alpha\,\dot{\phi}\,\dot{\tilde{\mathbf{B}}} + \beta\,e^{\alpha\phi}\,\dot{\xi}\,\nabla\times\tilde{\mathbf{B}}$$

$$(b) \quad \Box\tilde{\mathbf{E}} \equiv \ddot{\tilde{\mathbf{E}}} - \nabla^2\tilde{\mathbf{E}} = \alpha\left(\dot{\phi}\,\dot{\tilde{\mathbf{E}}} + \ddot{\phi}\,\tilde{\mathbf{E}}\right) + \beta\,e^{\alpha\phi}\left[\dot{\xi}\,\nabla\times\tilde{\mathbf{E}} - \left(\ddot{\xi} + \alpha\dot{\phi}\dot{\xi}\right)\tilde{\mathbf{B}}\right]. \tag{27}$$

We assume general wave solutions of the form

$$\tilde{\mathbf{B}}(\eta, \mathbf{x}) = \tilde{\mathbf{B}}_0(\eta)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\ ; \qquad \tilde{\mathbf{E}}(\eta, \mathbf{x}) = \tilde{\mathbf{E}}_0(\eta)\, e^{-i\mathbf{k}\cdot\mathbf{x}}. \tag{28}$$

Taking the z-direction as the propagation direction of the electromagnetic waves, i.e., $\mathbf{k} = k\hat{\mathbf{e}}_{\mathbf{z}}$, the equations for the polarization states, viz.,

$$b_{\pm}(\eta) = \tilde{B}_{0x}(\eta) \pm i\,\tilde{B}_{0y}(\eta)\ ; \qquad e_{\pm}(\eta) = \tilde{E}_{0x}(\eta) \pm i\,\tilde{E}_{0y}(\eta) \tag{29}$$

can be rewritten as

$$(a) \quad \ddot{b}_{\pm} - \alpha\,\dot{\phi}\,\dot{b}_{\pm} + \left(k^2 \mp k\beta e^{\alpha\phi}\dot{\xi}\right) b_{\pm} = 0$$

$$(b) \quad \ddot{e}_{\pm} - \alpha\,\dot{\phi}\,\dot{e}_{\pm} - \alpha\,\ddot{\phi}\,e_{\pm} + \left(k^2 \mp k\beta e^{\alpha\phi}\dot{\xi}\right) e_{\pm} = -\beta\,e^{\alpha\phi}\left(\alpha\dot{\phi}\dot{\xi} + \ddot{\xi}\right) b_{\pm}. \tag{30}$$

Now, to obtain the amount of rotation of the plane of polarization of electromagnetic radiation, given by the difference between the phases of the b_+ and b_- waves (or between those of the e_+ and e_- waves), we solve the above equations (30) separately for the different values of the parameters α and β corresponding to the compactification schemes I and II described in section 2. In what follows, we make an estimate of the optical rotation angle for both the schemes, first in a flat spacetime background and followed by a spatially flat background of FRW type.

4.1 Flat spacetime background

In order to make a preliminary idea as to how the coupling of KR axion and also the dilaton with Einstein-Maxwell theory affects the electromagnetic waves, and thereby results into optical activity in the radiation coming from distant galactic sources, we first consider the simplest situation — that is, of a flat Universe with cosmological scale factor $a = 1$ and η identified as the usual (cosmic) time coordinate t.

Case I : $\alpha = 1,\ \beta = 1$

In this case, which corresponds to the four-dimensional effective theory obtained through the Kaluza-Klein dimensional reduction of a higher dimensional EMKRD action, the solutions (26) for the axion and dilaton fields take the form:

$$\xi(t) = \tanh(ht)\ ; \qquad \phi(t) = \ln[\cosh(ht)] \tag{31}$$

while the equations (30) for the polarization states reduce to

$$(a) \quad \ddot{b}_{\pm} - h\,\tanh(ht)\,\dot{b}_{\pm} + \left[k^2 \mp kh\,\mathrm{sech}(ht)\right] b_{\pm} = 0$$

$$(b) \quad \ddot{e}_{\pm} - h\,\tanh(ht)\,\dot{e}_{\pm} - h^2\mathrm{sech}^2(ht)\,e_{\pm} + \left[k^2 \mp kh\,\mathrm{sech}(ht)\right] e_{\pm} = h^2\mathrm{sech}(ht)\tanh(ht)\,b_{\pm}.$$

$$\tag{32}$$

In general, it is very difficult to solve these equations exactly. However, while studying the KR field induced cosmic electromagnetic effects and the influence of the dilaton, we consider that the KR axion and the dilaton perturb the Maxwell's equations by a small amount in a standard FRW cosmological background with both matter and radiation dominant. Thus the KR field as well as the dilaton is assumed to have a negligible effect on shaping the cosmological background spacetime. One way of thinking about this is to imagine that the KR axion and the dilaton decouple from the radiation or matter (dust) fluid, shaping the cosmic geometry, far prior to dust-photon decoupling, leaving behind a 'cosmic axion-dilaton background' which affects incoming radiation from distant galaxies albeit rather softly. The electromagnetic effects produced as such will no doubt gradually subside, as the Universe expands further, however, may yet be observable in this epoch. In view of the smallness of such electromagnetic effects induced in axion-dilaton background, we consider the parameter h, which has the dimension of inverse time, to be sufficiently smaller than the wave number k of the electromagnetic waves. Assuming the standard phase-exponential ansatz of WKB type for the polarization states $b_\pm$ and $e_\pm$:

$$(a) \quad b_\pm(t) \;=\; b_0 \, e^{ikS_b^\pm(t)} \; ; \qquad S_b^\pm(t) = S_{b0}^\pm(t) + \frac{S_{b1}^\pm(t)}{k} + \frac{S_{b2}^\pm(t)}{k^2} + \cdots ,$$

$$(b) \quad e_\pm(t) \;=\; e_0 \, e^{ikS_e^\pm(t)} \; ; \qquad S_e^\pm(t) = S_{e0}^\pm(t) + \frac{S_{e1}^\pm(t)}{k} + \frac{S_{e2}^\pm(t)}{k^2} + \cdots \qquad (33)$$

with constant amplitudes b_0 and e_0, we obtain

$$S_b^\pm(t) \;\approx\; S_e^\pm(t) \;=\; t \;-\; \frac{1}{k}\left[\frac{i}{2} \ln\{\cosh(ht)\} \;\pm\; \tan^{-1}\left\{\tanh\left(\frac{ht}{2}\right)\right\}\right] + \mathcal{O}\left(\frac{1}{k^2}\right) \qquad (34)$$

Dropping the $\mathcal{O}(k^{-2})$ terms in the above expression, we finally obtain

$$b_\pm(t) \;=\; b_0 \sqrt{\cosh(ht)} \, \exp\left[ik\left(t \;\mp\; \frac{1}{k}\tan^{-1}\left\{\tanh\left(\frac{ht}{2}\right)\right\}\right)\right] \qquad (35)$$

and a similar expression for $e_\pm$. Thus, in presence of the KR axion and the dilaton there is a modulation of the amplitude as well as an alteration of the phase of free electromagnetic waves even in a flat spacetime background. The measure of the optical rotation angle can be obtained from the phase difference

$$\Delta(t_F) \;=\; |\arg f_+ \;-\; \arg f_-|_{t=t_F} \;=\; 2\tan^{-1}\left[\tanh\left(\frac{ht_F}{2}\right)\right] \qquad (36)$$

where $f_\pm$ is either $b_\pm$ or $e_\pm$, and t_F is the time interval between the emission and reception of the electromagnetic radiation in a flat background. For $ht_F < 1$ the above expression can be expanded as

$$\Delta(t_F) \;=\; ht_F \;+\; \frac{(ht_F)^3}{6} \;+\; \mathcal{O}(ht_F)^5. \qquad (37)$$

Let us now compare the results obtained here with that obtained in ref.[18] in which the effect of the axion (and not the dilaton) on the electromagnetic waves has been studied. In a flat spacetime background, the axion has been found to produce an optical rotation given by the amount ht_F upto

$\mathcal{O}(k^{-2})$, where h determines the rate of evolution of the KR axion [18]. As is evident from the expression (36) or (37), the dilaton produces an additional rotation given by an amount at most $\sim h^3 t_F^3$ (for $h t_F < 1$). Moreover, the amplitude modulation of the free electromagnetic waves discussed above is entirely due to the dilaton as the axion alone cannot produce such an effect.

Case II : $\alpha = \sqrt{\kappa r_c} e^{-\kappa r_c \pi} \ll 1, \ \beta = \sqrt{\frac{\kappa}{m_p}} e^{\kappa r_c \pi} \gg 1$

In this case, which corresponds to the four-dimensional effective theory obtained via RS compactification of a higher dimensional EMKRD action, the exponential dilaton coupling with the KR axion as well as with the electromagnetic field can be approximated to be ~ 1 as the parameter $\alpha \sim 10^{-16}$ for $\kappa r_c = 12$. On the other hand, the KR–electromagnetic coupling is enormously enhanced through the parameter β which is $\sim 10^{15}$ for $\kappa r_c = 12$. We consider the value of the parameter α to be zero for all practical purposes, whence in a flat background the solution for the axion is simply given by $\xi = ht$, and the equations for the polarization states take the form

$$\ddot{f}_\pm + \left(k^2 \mp k\beta h \right) f_\pm = 0 \tag{38}$$

where $f_\pm$ is either $b_\pm$ or $e_\pm$. The solution of this equation is given exactly as

$$f_\pm \sim \exp\left(i \sqrt{k^2 \mp k\beta h} \ t \right). \tag{39}$$

which implies that the angle of rotation of the plane of polarization of the electromagnetic waves given by

$$\Delta(t_F) = |\arg f_+ - \arg f_-|_{t=t_F} = kt_F \left(\sqrt{1 + \frac{\beta h}{k}} - \sqrt{1 - \frac{\beta h}{k}} \right). \tag{40}$$

Now, for $\kappa r_c = 12$, the value of $k/\beta = 2\pi e^{-\kappa r_c \pi} c/\lambda$ (where λ is the wavelength of the incoming radiation) which is typically $\sim 10^{-25}$ GeV for visible radiation and $\sim 10^{-31}$ GeV for radio waves. For $h < k/\beta$, the above expression for the optical rotation angle is given by

$$\Delta(t_F) = \beta h t_F + (\beta h)^2 \frac{t_F}{8k} + \cdots. \tag{41}$$

4.2 Spatially flat FRW spacetime background

Let us now consider a spatially flat expanding FRW background spacetime. Setting the present value of the cosmological scale factor to unity, one can express the latter in terms of the redshift z as

$$a = \frac{1}{(1 + z)}. \tag{42}$$

The flat space expressions for the optical rotation angle Δ given in the previous subsection 4.1, for the two cases I and II, can be generalized in a spatially flat Universe by replacing the time interval t_F in

Eqs.(36) and (40) by the cosmic lookback time t_L which is given in terms of redshift z as

$$t_L(z) = \int_0^z \frac{dz'}{(1+z') H(z')} \tag{43}$$

where $H = a^{-1}(da/dt)$ is the Hubble parameter. In a spatially flat FRW Universe, the lookback time t_L is the same as the distance traveled along light path from source to observer.

In what follows, we consider the standard spatially flat cosmological model where both (baryonic plus dark) matter and dark energy are dominant. The Hubble parameter is expressed as

$$H = H_0 \sqrt{\frac{\Omega_m + \Omega_d}{a^3} + \Omega_X} \tag{44}$$

where H_0 is the Hubble constant ($\sim 1.7 \times 10^{-18}$ s^{-1}); and Ω_b, Ω_d and Ω_X are the present values of the density parameters for the ordinary baryonic matter, dark matter and dark energy respectively. The total density parameter $\Omega = \rho/\rho_c = \Omega_b + \Omega_d + \Omega_\Lambda = 1$, where ρ is the total matter-energy density of the Universe and $\rho_c = 3H_0^2/(8\pi G)$ is critical density ($\sim 5 \times 10^{-27}$ Kg m^{-3}). The present accepted values of Ω_b, Ω_d and Ω_X are close to $0.05, 0.25$ and 0.7 respectively. The expression (43) for the lookback time reduces to

$$t_L(z) = \frac{1}{3H_0\Omega_X} \left[\ln \left(\frac{\sqrt{(\Omega_b + \Omega_d)(1+z)^3 + \Omega_X} - \Omega_X}{\sqrt{(\Omega_b + \Omega_d)(1+z)^3 + \Omega_X} + \Omega_X} \right) - \ln \left(\frac{1 - \Omega_X}{1 + \Omega_X} \right) \right]. \tag{45}$$

We estimate the value of the optical rotation angle Δ resorting separately to the two cases I and II corresponding to the two schemes of compactification and ascertain the maximum limit on the parameter h and hence on the effective KR field energy density ρ_{KR} from the observational bounds on the optical rotation angle [5, 28, 4].

Catalogues of observational data for radio galaxies and quasars [29] at distances comparable to the Hubble length $(= 1/H_0 \sim 10^{10}$ y) have been analyzed in great detail in [5, 28] for redshift $z \geq 0.4$. The observable of interest is the angular separation $|\chi - \psi|$, where ψ is the orientation angle of the axis of the radio source; and χ is the observed polarization angle extracted from the data on separating out the usual effect of Faraday rotation that occurs due to the passage of radiation through galactic/inter-galactic magnetized plasma. The Faraday rotation effect, which is proportional to the inverse square of the wave number k of the incoming radiation, is removed from the data by making a straight line fit of the total polarization angle θ as: $\theta = fk^{-2} + \chi$, where f is a Faraday rotation measure that depends on the plasma density and the parallel component of the background magnetic field along the direction of propagation of the wave. We refer to the upper bound on the observed amount of k-independent optical rotation angle obtained in [28] by analyzing the data for values of $|\chi - \psi|$ in the range $45^o - 135^o$:

$$\Delta_{max}^{(obs)} = 6^o \qquad \text{at redshift } z = 0.4 . \tag{46}$$

Case I : $\alpha = 1,\ \beta = 1$

On replacing the time interval t_F by the lookback time $t_L(z)$ [Eq.(45)] in the expression (36) for Δ (obtained in the flat space analysis for the case $\alpha = \beta = 1$),we have the optical rotation angle for the spatially flat Universe given by

$$\Delta(z) \;=\; 2\tan^{-1}\left[\tanh\left(\frac{ht_L(z)}{2}\right)\right] \tag{47}$$

with $t_L(z)$ as given in Eq.(45). Using the above upper bound ($\sim 6^o$) on Δ at $z = 0.4$, the maximum upper limit on the parameter h can be obtained: $h_{max}^{(obs)} = 5 \times 10^{-42}$ GeV . This implies that the effective KR field energy density which is given by

$$\rho_{KR} \;=\; \frac{M_p^2}{2a^2}\, e^{4\phi}\, \dot{\xi}^2 \;=\; \frac{M_p^2 h^2 (1+z)^2}{2} \tag{48}$$

is observationally limited to a maximum value $\rho_{KR}^{max} = 2 \times 10^{-46}$ GeV^4 at $z = 0.4$. Here we have used the relation $a = (1+z)^{-1}$ and the solutions (31), corresponding to the case $\alpha = \beta = 1$, for the axion and dilaton fields. This value of ρ_{KR}^{max} is sufficiently larger than the Cosmic Microwave Background (CMB) energy density $\rho_{cmb} \sim 4 \times 10^{-52}$ GeV^4 (corresponding to the temperature $T_{cmb} = 2.7$ K) but only two order larger than the dark matter energy density $\rho_d = \rho_c \Omega_d \sim 1.6 \times 10^{-48}$ GeV^4 . Lack of direct experimental support in favour of such a high background KR energy density, however, imposes a severe constraint on the value of ρ_{KR}^{max}. If we take ρ_{KR}^{max} to be at most of the order of ρ_{cmb}, then the maximum value of the parameter h is practically reduced to $h_{max}^{(cmb)} = 6.7 \times 10^{-45}$ GeV, which implies a substantial reduction in the maximum optical rotation angle $\Delta \sim 25$ arcsec at $z = 0.4$. On the other hand, if ρ_{KR}^{max} is taken to be $\sim \rho_{dm}$, then the maximum value of h is $h_{max}^{(dm)} = 1.3 \times 10^{-43}$ GeV, which implies a fairly larger maximum limit on $\Delta \sim 0.2^o$. In the second case one may hope for a possible measurement of such an effect with the improvement of observational accuracy.

It should now be mentioned here that for all the three upper bounds on h, viz., $h_{max}^{(obs)} \sim 10^{-42}$ GeV, $h_{max}^{(dm)} \sim 10^{-43}$ GeV and $h_{max}^{(cmb)} \sim 10^{-45}$ GeV, the contribution of the dilaton on the amount of optical rotation is extremely small, being at most cubic in $ht_L(z)$. For $h = h_{max}^{(obs)}$, the maximum change produced on Δ by the dilaton is $\delta\Delta \sim 0.05^o$, for $h = h_{max}^{(dm)}$, the dilaton's contribution is extremely insignificant $\delta\Delta \sim 0.003$ arcsec, while for $h = h_{max}^{(cmb)}$, the dilaton practically inflicts no change.

Case II : $\alpha = \sqrt{\kappa r_c}\, e^{-\kappa r_c \pi} \ll 1,\ \beta = \sqrt{\frac{\kappa}{m_p}}\, e^{\kappa r_c \pi} \gg 1$

Substituting the time interval t_F by the lookback time $t_L(z)$ in the flat space expression (40) for Δ, the optical rotation angle for the spatially flat FRW Universe can be given in terms of the redshift z as

$$\Delta(z) \;=\; k\left(\sqrt{1 + \frac{\beta h}{k}} - \sqrt{1 - \frac{\beta h}{k}}\right) t_L(z) \tag{49}$$

where $t_L(z)$ is as given in Eq.(45).

The maximum observational bound for Δ [Eq.(46)] sets a new upper limit on the parameter h at $z = 0.4$: $\quad h_{max}^{(obs)} = 2 \times 10^{-58}$ GeV . As such at $z = 0.4$, the effective KR field energy density is now limited to a extremely low maximum value $\rho_{KR}^{max} = 4 \times 10^{-79}$ GeV^4.This implies an unnatural fine-tuning of the KR energy density to a incredibly small value in order to avoid a catastrophically large wavelength independent rotation of the plane of polarization hitherto unobserved.

5   Conclusion

In this paper we have shown that a string inspired SUGRA model can offer a possible explanation for the hitherto unexplained wavelength independent optical rotation observed in the distant galactic radio waves. Among various possible sources one of the possible source of such a rotation is known to be the coupling between a pseudo-scalar and electromagnetic field. Such a coupling term was originally proposed from a phenomenological point of view by Carroll *et al* [5, 28] and considered also by [4, 30, 31, 32].However the coupling parameter in these models were arbitrary and needed to be determined either from experimental data or from some more fundamental theoretical framework where such a term occurs naturally. It turned out that the gauge invariant axion-em coupling originating in the context of anomaly cancellation in a string based SUGRA model exactly yields the desired coupling term. However the exact value of the coupling parameter depends crucially on the type of compactification used to compactify the extra dimensions.We have first shown that the other scalar field in such models, namely the dilaton, doesn't play any significant role in the phenomenon of optical rotation. Analysing the usual dilaton-axion-em coupled theory in four dimension , we found that despite a low value of the KR energy density, we can get a near measurable value of the optical rotation angle because of the large cosmological distance through which the em wave travels across the space. We have estimated an upper bound of this KR axion induced rotation by comparing the KR energy density with the known CMB and dark matter energy density. This whole analysis is subsequently carried out in the context of a braneworld scenario. The value of the coupling parameter in effective four dimensional theory , which depends on the type of compactification, gets enhanced by a large warp factor in RS scenario.This results into an unacceptably large value of the rotation of the plane of polarization perhaps indicating an inherent conflict between string inspired models and Randall-Sundrum scenario. It has been shown in a recent work [33] that a similar enhancement takes place for gravitational waves also. Some more exhaustive studies to explore this apparent conflict, are currently being carried out by the authors. It can be easily seen by extending our analysis that in the case of ADD compactification, no such large enhancement of the KR-electromagnetic coupling would take place which may lead to a high value of the rotation angle as found in RS scenario. Thus from this cosmological observation, it appears that ADD compactification is more compatible with string inspired

models than the RS compactification. This work therefore proposes a possibility to find a signature of the string inspired low energy model in astrophysical experiments.

Acknowledgment

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